

A COGNITIVE STUDY OF PROBLEM-SOLVING IN STATICS

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CONTEXT

The work described in this paper is part of an on-going study of problem solving in Statics (Van Meter et al., 2006; Higley, et al., 2007). The work is being done in Statics classes because it is one of the first places that engineering students encounter the engineering problem-solving process. In this study we are paying particular attention to the early steps in problem-solving when students ‘model’ the system being studied to create a set of equations describing the system. Clearly there are many different ways in which students can go wrong as they solve problems in Statics. Our working hypothesis is that students will cluster into different groups based on their abilities and knowledge, and that these groups will demonstrate differing abilities to solve Statics problems. Therefore, improving the problem-solving skills of these groups will require different interventions.

This study has been influenced by a number of studies of problem-solving in general and of problem-solving in engineering specifically. The relationship to past work was discussed at some length in a previous paper (Van Meter, et al., 2006) and therefore it is only briefly summarized here. Three subsets of the literature have had the most influence on our work: Problem-solving processes, translations between symbol systems, and domain knowledge.

Since seminal work by Polya (1957) in mathematics, the utility of learning and using a sequence of steps during problem-solving has been widely accepted. There are instructional interventions for engineering education that are grounded in this theoretical model of problem-solving, but the work of Woods may be best known. In his most recent work (Woods, 2000), Woods has focused on the processes of problem-solving and has developed a model to describe ideal problem-solving. Without a doubt, the quantity of prior domain knowledge affects problem-solving (Gelman and Greeno, 1989). The structure provided by the knowledge base can, for example, act as a constraint during analogical reasoning (Gentner, 1983), support strategic processing during reading (Alexander, et al. 1995), and contribute to positive motivational states during problem-solving (Hegarty-Hazel and Prosser, 1991). Within the domain of Statics, Paul Steif (2004) closely examined the role of misconceptions and developed a concept inventory in collaboration with Dantzler (Steif and Dantzler, 2005) to determine the effect of these misconceptions on problem-solving. A final approach to understanding problem-solving in engineering focuses on the symbol system translations inherent in the analysis process. By symbol system, we refer to the semiotic system used to understand and express elements and their relations. Translations are required when problem solvers move between symbol systems. McCracken and Newstetter (2001) developed the Text-Diagram-Symbol (TDS) model to capture the transformations that take place during analysis. The importance of visualization in transforming from a problem statement to a free-body diagram and the well documented gender effects on visualization skills (see for example, Sorby, 2001; Sorby and Baartmans, 2000; Devon, et al., 1998) led us to include spatial reasoning instruments in the study.

RESEARCH QUESTIONS

The current phase of the work is aimed at answering two main questions about the modeling processes:

- What are the major difficulties that students encounter when they perform modeling during problem-solving in Statics?
- What instructional interventions will address these problems and improve engineering students' modeling during problem-solving?

THEORETICAL FRAMEWORK

Prior work in the literature on problem solving has been synthesized to formulate an “integrated model of problem solving” (Van Meter et al., 2006) to guide this research. The model, which builds upon the model of Newstetter and McCracken, involves three recursive processes: representation, framing, and synthesis. During representation, the problem solver reads the problem and constructs a mental representation of the situation. Ideally the deep structure of the problem is identified and goals are set. In the framing phase, the solver creates a physical representation of the problem, in this case a free-body diagram, and also maps given information. In the final phase the emphasis shifts to mathematical formulation and solution.

METHODOLOGY

In order to identify clusters of students, data was collected on three types of measures: mathematics, spatial reasoning and conceptual knowledge related to Statics. Participants were offered extra credit on their course grade for the completion of the measures. During their first visit to the website, students read and indicated agreement with the informed consent and answered basic demographics questions, such as gender, race, SAT scores, major, and GPA; they were then brought to new page containing a separate link to each measure. The measures included a math test, two spatial reasoning tests, and Steif's Statics Concept Inventory. Items in the math test covered trig and vector concepts important to problem solving in Statics. Spatial reasoning was measured by two well-accepted measures in the field, Card Rotation and Paper Folding from the Factor-Referenced Cognitive Tests (Ekstrom et al., 1976). Both tests are timed, limiting the students to three minutes for each set of items (12 minutes total). Knowledge related to Statics was measured using the Statics Concept Inventory (Steif and Hansen, 2006), which is a 27-item measure of the concepts that have been identified as key in Statics comprehension. Ward's method of cluster analysis (Ward, 1963) was applied to the data to identify clusters whose members performed similarly on the measures. In this method, the squared Euclidean distances are the measure of the differences between the groups.

In the first round of the data collection, which took place during Fall 2006, the cluster analysis was followed by selection of students for think-aloud sessions. 39 students were randomly selected across the clusters to participate in one-on-one sessions that included think-aloud problem solving and discussion of items from the Statics Concept Inventory that were identified as discriminating well across the clusters. The think-aloud problems asked the students to create a free-body diagram and the corresponding set of equilibrium equations. In addition students were asked qualitative questions about their problem solving in Statics. Data from these interviews was analyzed by a team of six expert instructors to identify key difficulties across the clusters.

Based on the key difficulties that were identified, a cross-functional team of experts from engineering and educational psychology worked to create and refine two interventions through a

series of ‘design experiments’ in which the interventions were used, evaluated and improved. The goal of the design process was to create ‘materials-driven’ interventions that would be done by students outside of the classroom without interaction with the instructor or teaching assistant. In order to assess the effectiveness of the interventions, a short pre/post-test was created. This test was also refined throughout the series of design experiments.

Of the approximately 560 students enrolled in the class, 391 completed all three measures on the website. The majority of the participants were white (87%), male (85%), and sophomores (88%). A total of 233 students chose to participate in the design experiments process; all received extra course credit for participating. Some of these students, however, had not completed the inventories used in the cluster analysis. Therefore only 164 students completed the inventories and participated in the design experiments process.

FINDINGS AND CONCLUSIONS

Analysis of the videos of the think-aloud problem solving and discussion questions was done by the team of six experts, five faculty members in Engineering Mechanics/Mechanical Engineering and one Ph. D. candidate in Engineering Mechanics, who has been an instructor of mechanics courses for five years. This same team also scored the students’ written work from the sessions. A significant finding of the analysis was that the key difficulties that were identified were largely independent of the cluster of the student; contrary to our working hypothesis that difficulties would vary by cluster.

The major difficulties in problem solving identified through the analysis of the think-aloud videos and student work were:

- Students did not grasp fully the concept of a free-body diagram including the distinction between internal and external forces
- Students relied mostly on memory to decide what reactions to include based on the type of connection/interaction.
- Students did not have a physical understanding of the reactions that could be supported by different types of connections/interactions between bodies.
- Students often failed to include a moment equation in their equilibrium equation set.
- Some students had significant difficulty with trigonometry

Based on the results of the analysis of students’ work and the videos, the research team identified a number of possible interventions. The team decided to focus on interventions that required students to draw the reactions at given connection/interaction and to analyze a given free-body diagram and identify whether the reactions shown are correct or not. For both interventions, students were asked to explain their answers, which required them to undertake a process of ‘elaboration’ that has been identified as one approach to increasing conceptual understanding (Chi et al., 1994).

The justification statements typed by representative students from high and low performing clusters were reviewed by a two pairs of content experts. The scoring grid included six categories, four for incorrect justifications and two for correct justifications. A category of ‘effectively no explanation’ was used for a statement such as “there are both x and y-reactions because it is a pin connection.” Such answers did not reveal anything about the student’s understanding of the motions permitted by a connection and their relationship to the possible reactions. This type of answer indicates that students were working from memory rather than reasoning. Other categories in the scoring grid included answers that were incorrect because the student mistakenly included effects of loading or other connections in their justification. A third

category of incorrect arguments was those based on possible motions but which were offered in support of an incorrect answer. The overall fraction of correct responses in the high and low performing groups was approximately the same, 72% versus 60%. However, students in the high performing clusters were much more likely to provide correct reasoning and much less likely (60% vs. 29%) to offer justifications that suggested that they were relying on memory.

The five item pre-post test, also developed through the design experiments process, included two questions on reactions, two questions on free-body diagrams and one on a moment equation. In the final round of design experiments, a list of justifications that included distracters based on student responses in second round of the design experiments was added to Question 5 of the pre/post-test; the figure and answers for Q5 are shown in Figure 1.

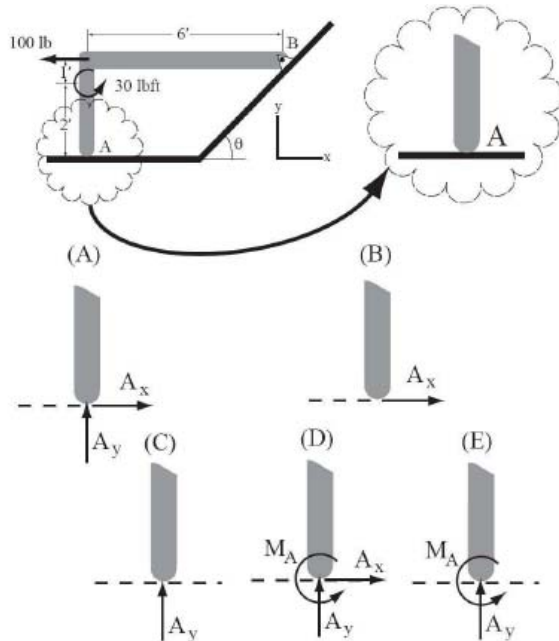


Figure 1. Question 5 on the pre/post-test

the potential to improve student performance. Work is underway to increase the number of items in the interventions and to enhance the reliability of the pre/post-test prior to full scale testing of the interventions in Fall 2008.

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The performance of the students on Q5 was disappointing for both the pre-test and post-test. Out of the 215 students who gave meaningful responses to Q5, 161 selected the correct free-body diagram, but of those only 29 selected the correct justifications for all three of the possible reactions. On the post-test, the number of students who selected the correct FBD was essentially unchanged (159), but the number who selected all of the correct reasons increased to 38.

RECOMMENDATIONS

Although the work to date cannot support recommendations because of the small numbers of students who improved after completing the interventions, it does indicate that the design experiments process was successful in developing materials-driven interventions with

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